Time periodicity condition of magnetostatic problem coupling with electric circuit imposed by Waveform Relaxation Method

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In numerical computation, the finite element (FE) method associated with external electric circuits is often used to evaluate electromagnetism devices with voltage sources. To study the solution of the steady state, the computation time can be prohibitive due to a large transient state compared with the time step used to discretize the time domain. In this communication, the Waveform Relaxation Method is developed to impose the steady state of the solution in the case of magnetostatic problem coupled with electric cricuit equation.

***Index Terms*—finite element method, magnetostatic problem, Waveform relaxation method**

I. Introduction

DURING the last ten years, in the frame of electromagnetism, a large number of time-stepping finite-element (FE) model have been proposed. Usually, conventional models solve problems in the time domain with implicit euler method. These models are efficient tools to simulate, as example, magnetostatic or magnetodynamic problems coupling with electric circuits [1]. If the constant time of studied device is large compared with the time step used to discretize the time domain, the computation time can be prohibitive to obtain the steady state. In this context, some methods have been proposed in the literature to ensure the steady state and to reduce the computation time. In [2], authors developed a method based on the harmonic-balance approach. In [3], a time step method based on the Fourier decomposition is presented but the convergence rate depends on the application example.

In this communication, we propose to use the so-called dynamic iteration or waveform relaxation method (WRM) [4], to impose the steady state of the solution without transient state. The WRM approach enables to couple FE models and circuit equations. Many contributions exist, applied to circuits but also to electric networks, mechanical problems, or circuit-FE model coupling [5].

First, the numerical model obtained from the vector potential formulation used to solve magnetostatic problem coupled with electric circuit is briefly presented. Secondly, the WRM approach is developed in order to impose a time periodicity condition on the solution and then the steady state. Finally, to show the efficiency of the proposed method, an academic example based on a simple electromagnetic structure supplied by voltage is studied.

II. Numerical approach

A. Magnetostatic problem coupling with electric circuit

We consider a magnetic core and an inductor connected to a voltage supply. The waveform of the voltage is periodic with a fundamental frequency f and a period $T = 1/f$. This problem can be solved by the differential equation $v(t) = Ri(t) + \frac{d\phi(t)}{dt}$ with $v(t)$ the voltage, R the resistance of the winding, $i(t)$ the current and $\phi(t)$ the linkage flux. To calculate the linkage flux which depends on the parameters of the structure, a magnetostatic problem can be considered. To solve this kind of problem, the magnetic vector potential formulation is used. Then, the partial differential equation to be solved is $\mathbf{curl}\,(\nu\ \mathbf{curl}\ \mathbf{A}(\mathbf{x},t)) = \mathbf{N}(\mathbf{x})i(t)$ on $D \times [0, mT]$, $m \in \mathbb{N}$, with $\mathbf{A}$ the vector potential, ν the magnetic reluctivity which is supposed constant (linear problem) and $\mathbf{N}$ the unit current density vector depending on the geometry of the inductor. The linkage flux can be expressed by $\phi(t) = \int \mathbf{A}(\mathbf{x},t)\mathbf{N}(\mathbf{x})dD$ to couple the electric and magnetostatic equations. The space discretization is carried out by the FE method with Whitney's edges element, while the time discretization with a constant time step $\Delta t = T/N_t$ with N_t the number of interval per period. Finally, the equations at the j^{th} time step are:

$$v_j = Ri_j + \mathbf{F}^t \frac{\mathbf{X}_j - \mathbf{X}_{j-1}}{\Delta t} \quad (1)$$

$$\mathbf{M}\ \mathbf{X}_j = \mathbf{F}i_j \quad (2)$$

With $\mathbf{X}$ the solution vector corresponding to the circulation of $\mathbf{A}$ on all edges of the mesh, $\mathbf{F}$ a vector, $\mathbf{M}$ a sparse matrix depend on the magnetic reluctivity, $i_j = i(j\Delta t)$, $v_j = v(j\Delta t)$, $\mathbf{X}_j = \mathbf{X}(j\Delta t)$ and $j \in \{1, 2, \ldots, N_t, \ldots\}$.

B. Imposition of time periodicity condition with WRM

To solve the system of equations defined by ((1),(2)), the WRM is used [4][5]. This approach is based on a fixed point technique. At the k^{th} iteration, firstly, the equation (1) is solved on $[0, mT]$, $m \in \mathbb{N}$, with the solution vectors $\mathbf{X}_j^{k-1}$, $j = 1 \ldots mN_t$, computed at the previous iteration. Then, we obtain the values of the current on all time steps. Secondly, the equation (2) is solved on $[0, mT]$ and we obtain the solution vectors $\mathbf{X}_j^k$ on all time steps. Finally, these two steps are repeated until convergence. To impose the steady state of the current and the vector potential, considering one period, we ensure the time periodicity condition $\mathbf{X}_0 = \mathbf{X}_{N_T}$ and $i_0 = i_{N_T}$. As the developed approach is based on a fixed point technique, the convergence depends on the initial condition [4]. To ensure the convergence, we introduce new term, such as $\phi(t) = L_f i(t) + \phi_r(t)$ with ϕ_r a residual term and L_f the value of an inductance given by the user. This approach is similar to the under and over relaxation methods. The choice of L_f affects the convergence. In the case of simple geometry, it is possible to calculate analytically an approximation of this value. Then, at the k^{th} iteration of WRM, the equations to be solved on $[0, T]$ are:

$$Ri_j^k + \frac{L_f}{\Delta t} i_j^k = v_j + \frac{L_f}{\Delta t} i_{j-1}^k - \frac{1}{\Delta t}(\phi_{r_j}^{k-1} - \phi_{r_{j-1}}^{k-1}) \quad (3)$$

$$\mathbf{M}\ \mathbf{X}_j^k = \mathbf{F} i_j^k \quad (4)$$

At each iteration, the residual term is updated by $\phi_{r_j}^k = \phi_j^k - L_f i_j^k$ with $\phi_j^k = \mathbf{F}^t \mathbf{X}_j^k$.

III. Application

To evaluate the accuracy of the proposed method, an electromagnetic structure composed of a ferromagnetic core with an air-gap and a winding supplied by voltage is studied. The waveform of the voltage is $v(t) = sin(2\pi f j \Delta t)$ with $j \in \{1, \ldots, 40\}, f = 50hz$ and $\Delta t = 0.5ms$. The structure (Fig. 1) has been discretized on tetrahedron mesh (180.000 elements). Figure 2 presents the distribution of the magnetic flux density for a given value of the current.
Analytically, the value of the inductance is $L = 32.5mH$. With the classical approach based on time stepping FE model, the steady state is reached after $500ms$ which correspond at 25 periods. With this model, the evolution of the current is shown in Fig. 3 for 50 periods. To impose the steady state by ensuring time periodicity condition, only one period is considered and the system of equations ((3),(4)) is solved. To evaluate the convergence, the evolution of the relative error of the current between two successive iterations of the WRM is presented on Fig. 4 for different values of L_f. We can observe that the value of L_f influences significantly the convergence, if L_f is under the real value, the convergence is faster but less stable. Figure 5 shows the evolution of the current for one period obtained from the time stepping FE model (Reference) at the steady state and this one computed from the proposed method at different iterations of the WRM. With 16 iterations of the WRM, the evolution of the current is close to the reference. With this application example, the computation time has been reduced about 30%. It can be noted that the method can be extended to the non-linear case.

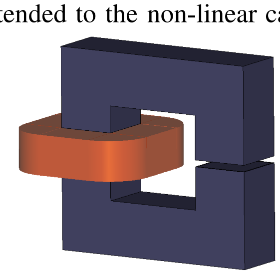

Fig. 1. Magnetic studied structure

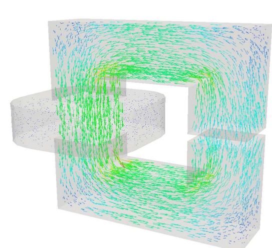

Fig. 2. magnetic flux density distribution

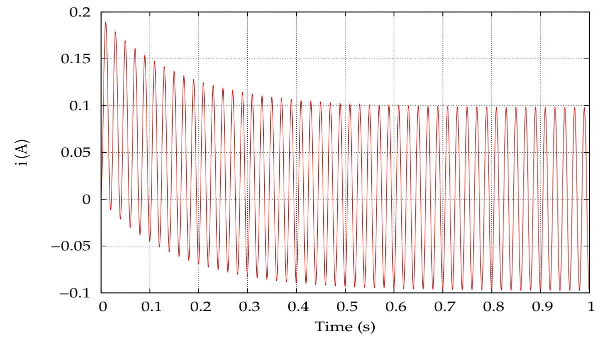

Fig. 3. Evolution of the current obtained by time stepping FE model on 50 periods

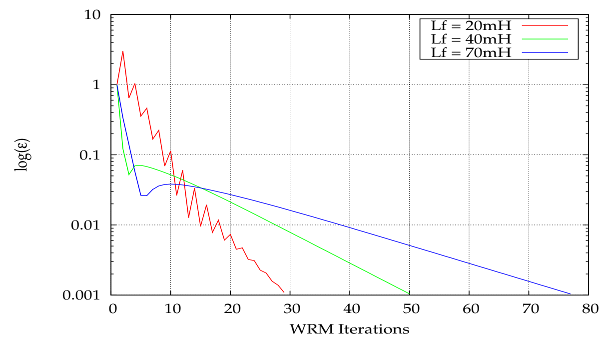

Fig. 4. Relative error of the current for different value of L_f

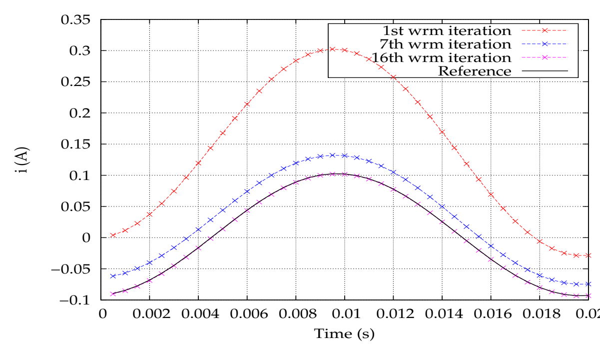

Fig. 5. Evolution of the current at the steady state obtained from the time stepping FE model and from the WRM at different iterations with $L_f = 20mH$

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